a vacuum spark can reach several tens of amperes with a discharge duration of 20-50 nsec; during the discharge there is intense vaporization of the electrode material, whose vapor is subsequently ionized. The main reason for the irreproducibility is the spread of the electrical parameters of the discharge, which determine the energy input to the spark and, consequently, the characteristics of the plasma produced: Stabilization of the discharge current and voltage of individual breakdowns should lead to an improvement in the metrological characteristics of instruments.

NOTATION

C, capacitance; R, resistance; L_S, stray inductance; I, current; U, voltage; L_λ, emission intensity.

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THEORY OF THE POSITIVE COLUMN OF A NONSTEADY ELECTRIC ARC IN A CHANNEL CONTAINING A GAS STREAM

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The problem of a nonsteady arc column in a channel containing a gas stream is solved for an arbitrary law of variation of the current with allowance for the variability of the gas density, flow velocity, and flow rate.

Theoretical investigations of the interaction of the positive column of a nonsteady electric arc in a channel containing a gas stream are associated with considerable difficulties. Analytical solutions of the problem can be obtained only by adopting a number of simplifying assumptions. In [1], for example, the problem was solved when convective energy losses can be neglected, while in [2] the gas flow rate was constant. As a result of the heating of the gas in the electric arc, however, its velocity and flow rate can vary considerably along the channel [3, 4]. Therefore, for a fuller understanding of processes of interaction of an arc discharge with a gas stream, one must allow for the variability of its velocity and flow rate.

It is assumed that the entire channel is filled with an electrically conducting gas, while the main role in the energy balance of the arc is played by processes of Joule dissipation of the energy of the electric field, heat transfer in the radial direction due to heat conduction, convective energy transfer along the channel axis, and emission, and the properties of the positive column of a nonsteady arc in a gas stream can be described by the equations

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$$\rho\omega\frac{dW}{dt}+\rho v\frac{\partial W}{l\partial z}=\frac{1}{R^2r}\frac{\partial}{\partial r}\left(r\frac{\partial S}{\partial r}\right)+\sigma E^2-\varepsilon, \tag{1}$$

$$\frac{\partial\rho}{\partial t}+\frac{1}{l\omega}\frac{\partial}{\partial z}(\rho v)=0, \tag{2}$$

$$I(t)=2\pi R^2E(z,\ t)\int_0^1\sigma(r,\ z,\ t)\,rdr. \tag{3}$$

We introduce the function $N = \rho W$, and then in a linear approximation of the dependences of N, σ, and ε on S, from (1)-(3) we obtain

$$\frac{\partial S}{\partial t}+\frac{v}{l\omega}\frac{\partial S}{\partial z}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial S}{\partial r}\right)+c_1E^2S-\Omega S, \tag{4}$$

$$I(t)=2\pi R^2\sigma_sE(z,\ t)\int_0^1 S(r,\ z,\ t)\,rdr. \tag{5}$$

Here

$$a=\frac{1}{R^2N_s\omega};\ c_1=\frac{\sigma_s}{N_s\omega};\ \Omega=c_2+\frac{1}{l\omega}\frac{\partial v}{\partial z};\ c_2=\frac{\varepsilon_s}{N_s\omega};$$

$$N_s=\frac{\partial N}{\partial S};\ \sigma_s=\frac{\partial\sigma}{\partial S},\ \varepsilon_s=\frac{\partial\varepsilon}{\partial S}.$$

We will solve Eqs. (4)-(5) with the conditions

$$S(r,\ 0,\ t)=\varphi_1(r,\ t),\ S(r,\ z,\ 0)=\varphi_2(r,\ z),\ S(1,\ z,\ t)=S_r(0,\ z,\ t)=0,$$
$$v=v_0\alpha(z),\ \alpha(z)=1+kz,\ \alpha>0. \tag{6}$$

Despite the simplifying assumptions, in contrast to [1, 2] and others, the present problem makes additional allowance for the variability of the density, flow velocity, and flow rate of the gas.

We will seek the solution of the nonlinear equation (4) with an arbitrary law of variation of E(z, t). Introducing the new independent variables

$$r_1=r,\ x=z,\ \tau=t-\frac{1}{b\cdot k}\ln(1+kz),\ \left(b=\frac{v_0}{l\omega}\right), \tag{7}$$

from (4) we obtain

$$b(1+kx)\frac{\partial S}{\partial x}=\frac{a\partial}{r_1\partial r_1}\left(r_1\frac{\partial S}{\partial r_1}\right)+c_1E^2(x,\ \tau)S-\Omega S. \tag{8}$$

We represent $S(r_1, x, \tau)$ in the form

$$S(r_1,\ x,\ \tau)=\exp\left(\frac{c_1}{b}\int_0^x\frac{E^2(x,\ \tau)}{1+kx}dx\right)U(r_1,\ x,\ \tau). \tag{9}$$

The substitution of (9) into (8) gives

$$b(1+kx)\frac{\partial U}{\partial x}=\frac{a}{r_1}\frac{\partial}{\partial r_1}\left(r_1\frac{\partial U}{\partial r_1}\right)-\Omega U. \tag{10}$$

Equation (10) is linear. Solving it by the Fourier method, substituting the result into (9), and returning to the variables r, z, and t, we find

$$S(r,\ z,\ t)=\exp\left(\frac{c_1}{b}\int_0^z\frac{E^2dx}{1+kx}\right)\sum_{n=1}^{\infty}f_n(\tau)\,\alpha^{-\frac{\beta_n+k}{k}}J_0(\lambda_nr), \tag{11}$$

where

$$\beta_n=\frac{\lambda_n^2a+c_2}{b};\ f_n(\tau)=f_n\left[t-\frac{1}{b\cdot k}\ln(1+kz)\right].$$

We determine the arbitrary functions $f_n(\tau)$ from (11) and (6):

$$f_n(\tau) = \begin{cases} A_n(\tau) & \text{for } \tau > 0, \\ B_n(\psi)\Phi(\psi)\exp(-\tau b(\beta_n + k)) & \text{for } \tau < 0. \end{cases} \quad (12)$$

Here $A_n(\tau)$, $B_n(\psi)$, and $\Phi(\psi)$ are functions obtained through the transformation of $A_n(t)$, $B_n(z)$, and $\Phi(z)$, respectively, and $A_n(t)$ and $B_n(z)$ are Fourier coefficients in the series exapnsions of $\varphi_1(r, t)$ and $\varphi_2(r, z)$ in the functions $J_0(\lambda_n r)$ in the interval of $0 \leq r \leq 1$ satisfying the conditions $A_n(t) = 0$ at $\tau < 0$ and $B_n(z) = 0$ at $\tau > 0$.

$$\Phi(z) = \exp\left\{-\frac{I^2(0)c_1}{4\pi^2 R^4 \sigma_s^2 b}\int_0^z \frac{\left[\int_0^1 \varphi_2(r, x)\,r\,dr\right]^{-2}}{1+kx}dx\right\},$$
$$\psi = \frac{1}{k}[\exp(-bk\tau) - 1]. \quad (13)$$

From (11), (5), and (12) we obtain the general solution of Eq. (4):

$$S(r, z, t) = \frac{I(t)}{2\pi R^2\sigma_s E(z, t)}\left[\frac{\sum_{n=1}^{\infty} A_n(\tau)\alpha^{-\frac{\beta_n+k}{k}} J_0(\lambda_n r)}{\sum_{n=1}^{\infty} A_n(\tau)\alpha^{-\frac{\beta_n+k}{k}}\gamma_n} + \frac{\sum_{n=1}^{\infty} B_n(\psi)\exp(-bt(\beta_n+k))J_0(\lambda_n r)}{\sum_{n=1}^{\infty} B_n(\psi)\exp(-bt(\beta_n+k))\gamma_n}\right]. \quad (14)$$

To calculate the dependence $E(z, t)$ we have from (5) and (11) the equation

$$E(z, t)\exp\left(\frac{c_1}{b}\int_0^z \frac{E^2(x, \tau)}{1+kx}dx\right) = \Psi(z, t). \quad (15)$$

Integrating, we find

$$E(z, t) = \Psi(z, t)\Big/\left[1 + 2\frac{c_1}{b}\int_0^z \frac{\Psi^2(x, \tau)}{1+kx}dx\right]^{1/2},$$
$$\Psi(z, t) = I(t)/2\pi R^2\sigma_s \sum_{n=1}^{\infty} f_n(\tau)\alpha^{-\frac{\beta_n+k}{k}}\gamma_n,\quad \gamma_n = \frac{J_1(\lambda_n)}{\lambda_n}. \quad (16)$$

The expressions (14) and (16) are the exact solution of the system of Eqs. (4)-(5) with the conditions (6). The distributions $S(r, z, t)$ and $E(z, t)$ are obtained in the form of a superposition of plane waves traveling in the positive direction of the z axis. The variable τ has the meaning of the characteristic time, i.e., the time of observation of a wave when moving together with it with the propagation velocity. In the region of the positive column of the arc ahead of the wave front, the distributions of the local characteristics of the arc are determined by the function $\varphi_2(r, z)$. With developed nonsteadiness ($\tau > 0$), the influence of the initial distribution on the properties of the arc disappears, while the distributions $S(r, z, t)$ and $E(z, t)$ are determined by the function $\varphi_1(r, t)$.

These equations allow one to calculate the thermal and electrical characteristics of an arc in a channel containing a gas stream with an arbitrary law of variation of the current and with allowance for the variability of the flow velocity and flow rate.

To simplify the analysis of the solution obtained, let us consider particular cases.

1. Let $\varphi_1(r, t) = A_1 J_0(\lambda_1 r)$, $I(t) = I_0(1 + i\cos t)$, $i = I_m/I_0 < 1$, a case when small perturbations are imposed on a constant-current arc. For $\tau > 0$, from (14)-(16) we obtain

$$S(r, z, t) = I(t)J_0(\lambda_1 r)/2\pi R^2\sigma_s E(z, t)\gamma_1, \quad (17)$$

$$E(z, t) = E_\infty(1 + i\cos t)\alpha^{\frac{\beta_1+k}{k}}/M(z, t). \quad (18)$$

Here

$$M(z, t) = \left[\frac{E_\infty^2}{E_0^2} + \left(1+\frac{i^2}{2}\right)(\alpha^{2\frac{\beta_1+k}{k}} - 1)\frac{\beta_1}{\beta_1+k} + 4i\Theta + \frac{i^2}{2}F\right]^{1/2}; \quad (19)$$

$$\Theta(z, t) = \frac{\beta_1 b}{4b^2(\beta_1+k)^2+1}\left\{[2b(\beta_1+k)\cos t + \sin t]\,\alpha^{2\frac{\beta_1+k}{k}} - 2b(\beta_1+k)\cos\tau - \sin\tau\right\};$$

$$F(z,\ t) = \frac{\beta_1 b}{b^2(\beta_1+k)^2+1}\{[b(\beta_1+k)\cos 2t+\sin 2t]\,\alpha^{2\frac{\beta_1+k}{k}} - b(\beta_1+k)\cos 2\tau - \sin 2\tau\};\ \tau = t - \frac{1}{bk}\ln(1+kz);$$

$$E_0 = I_0/2\pi R^2\sigma_s A_1\gamma_1;\ E_\infty^2 = (\lambda_1^2+\varepsilon_s R^2)/\sigma_s R^2.$$

2. In nonsteady arcs with high-frequency current oscillations, the temperature and conductivity are unable to vary significantly over the oscillation period. In this case as $\omega \to \infty$ Eq. (17) has the form

$$S(r,\ z,\ t) = \frac{I_0 J_0(\lambda_1 r)}{2\pi R^2\sigma_s E_\infty\gamma_1}\,\alpha^{-\frac{\beta_1+k}{k}}\left[\frac{E_\infty^2}{E_0^2} + \frac{\beta_1}{\beta_1+k}\left(1+\frac{i^2}{2}\right)\left(\alpha^{2\frac{\beta_1+k}{k}}-1\right)\right]^{1/2}. \tag{20}$$

As is seen, S does not depend on time.

3. With an increase in the channel length and a decrease in the gas flow rate, the local characteristics of the arc tend toward limiting values. As $l/v_0 \to \infty$, from (18)-(19) we obtain

$$E(z,\ t) = [E_\infty(1+i\cos t)]/\left[1+\frac{i^2}{2}+\frac{4ib\beta_1}{4(b\beta_1)^2+1}(2b\beta_1\cos t+\sin t)+\frac{i^2 b\beta_1}{(b\beta_1)^2+1}(b\beta_1\cos 2t+\sin 2t\right]^{1/2}. \tag{21}$$

It is easy to obtain an equation for S(r, z, t) by substituting (21) into (17).

4. For k = 0 these equations agree with the results of [2], while for $\rho v = 0$ they coincide with the solution given in [1].

NOTATION

$S = S_1 - S_*$; $S_1 = \int_0^T \varkappa dT$, heat-conduction function; S_*, value of S_1 at the channel wall; R, l, channel radius and length; r, z, cylindrical coordinates, normalized to R and l, respectively; $t = t_1\omega$; t_1, time; ω, oscillation frequency; σ, $\varkappa$, W, ρ, ε, electrical conductivity, thermal conductivity, internal energy, density, and emissivity of the gas; I_0, I_m, constant component and amplitude of the variable component of the current; E, electric field strength; v, flow velocity; $J_0(\lambda_n r)$, zero-order Bessel function; λ_n, roots of the equation $J_0(\lambda_n 1) = 0$.

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